PDHonline Course E237 (3 PDH)

Electrical Fundamentals – Introduction to Inductance

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CHAPTER 2

INDUCTANCE

LEARNING OBJECTIVES

Upon completion of this chapter you will be able to:

1. Write the basic unit of and the symbol for inductance.
2. State the type of moving field used to generate an emf in a conductor.
3. Define the term "inductance."
4. State the meanings of the terms "induced emf" and "counter emf."
5. State Lenz's law.
6. State the effect that inductance has on steady direct current, and direct current that is changing in magnitude.
7. List five factors that affect the inductance of a coil, and state how various physical changes in these factors affect inductance.
8. State the principles and sequences involved in the buildup and decay of current in an LR series circuit.
9. Write the formula for computing one time constant in an LR series circuit.
10. Solve L/R time constant problems.
11. State the three types of power loss in an inductor.
12. Define the term "mutual inductance."
13. State the meaning of the term "coupled circuits."
14. State the meaning of the term "coefficient of coupling."
15. Given the inductance values of and the coefficient of coupling between two series-connected inductors, solve for mutual inductance, M.
16. Write the formula for the "total inductance" of two inductors connected in series-opposing.
17. Given the inductance values of and the mutual inductance value between two coils connected in series-aiding, solve for their combined inductance, L_T.

INDUCTANCE

The study of inductance presents a very challenging but rewarding segment of electricity. It is challenging in the sense that, at first, it will seem that new concepts are being introduced. You will realize as this chapter progresses that these "new concepts" are merely extensions and enlargements of fundamental principles that you learned previously in the study of magnetism and electron physics. The study of inductance is rewarding in the sense that a thorough understanding of it will enable you to acquire a working knowledge of electrical circuits more rapidly.

CHARACTERISTICS OF INDUCTANCE

Inductance is the characteristic of an electrical circuit that opposes the starting, stopping, or a change in value of current. The above statement is of such importance to the study of inductance that it bears repeating. Inductance is the characteristic of an electrical conductor that OPPOSES CHANGE in CURRENT. The symbol for inductance is L and the basic unit of inductance is the HENRY (H). One henry is equal to the inductance required to induce one volt in an inductor by a change of current of one ampere per second.

You do not have to look far to find a physical analogy of inductance. Anyone who has ever had to push a heavy load (wheelbarrow, car, etc.) is aware that it takes more work to start the load moving than it does to keep it moving. Once the load is moving, it is easier to keep the load moving than to stop it again. This is because the load possesses the property of INERTIA. Inertia is the characteristic of mass which opposes a CHANGE in velocity. Inductance has the same effect on current in an electrical circuit as inertia has on the movement of a mechanical object. It requires more energy to start or stop current than it does to keep it flowing.

Q1. What is the basic unit of inductance and the abbreviation for this unit?

ELECTROMOTIVE FORCE (EMF)

You have learned that an electromotive force is developed whenever there is relative motion between a magnetic field and a conductor.

Electromotive force is a difference of potential or voltage which exists between two points in an electrical circuit. In generators and inductors the emf is developed by the action between the magnetic field and the electrons in a conductor. This is shown in figure 2-1.

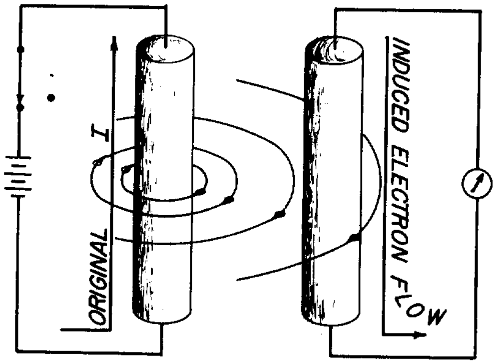
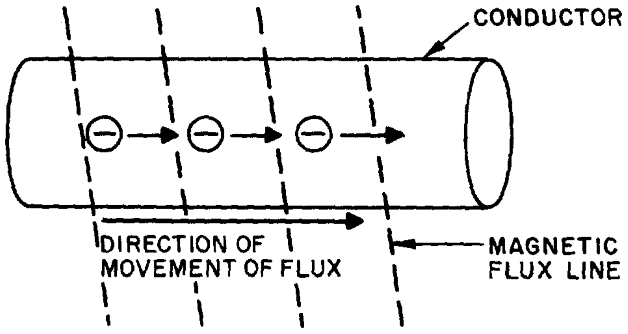
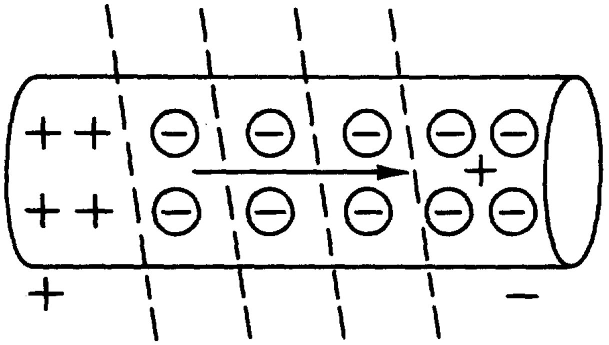


Figure 2-1.—Generation of an emf in an electrical conductor.

When a magnetic field moves through a stationary metallic conductor, electrons are dislodged from their orbits. The electrons move in a direction determined by the movement of the magnetic lines of flux. This is shown below:

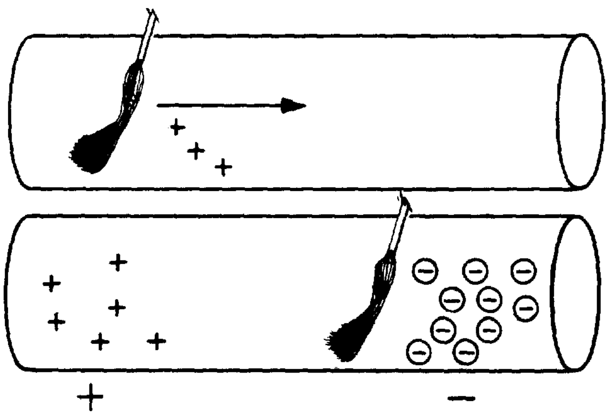


The electrons move from one area of the conductor into another area. The area that the electrons moved from has fewer negative charges (electrons) and becomes positively charged. The area the electrons move into becomes negatively charged. This is shown below:



The difference between the charges in the conductor is equal to a difference of potential (or voltage). This voltage caused by the moving magnetic field is called electromotive force (emf).

In simple terms, the action of a moving magnetic field on a conductor can be compared to the action of a broom. Consider the moving magnetic field to be a moving broom. As the magnetic broom moves along (through) the conductor, it gathers up and pushes electrons before it, as shown below:



The area from which electrons are moved becomes positively charged, while the area into which electrons are moved becomes negatively charged. The potential difference between these two areas is the electromotive force or emf.

Q2. An emf is generated in a conductor when the conductor is cut by what type of field?

SELF-INDUCTANCE

Even a perfectly straight length of conductor has some inductance. As you know, current in a conductor produces a magnetic field surrounding the conductor. When the current changes, the magnetic field changes. This causes relative motion between the magnetic field and the conductor, and an electromotive force (emf) is induced in the conductor. This emf is called a SELF-INDUCED EMF because it is induced in the conductor carrying the current. The emf produced by this moving magnetic field is also referred to as COUNTER ELECTROMOTIVE FORCE (cemf). The polarity of the counter electromotive force is in the opposite direction to the applied voltage of the conductor. The overall effect will be to oppose a change in current magnitude. This effect is summarized by Lenz's law which states that: THE INDUCED EMF IN ANY CIRCUIT IS ALWAYS IN A DIRECTION TO OPPOSE THE EFFECT THAT PRODUCED IT.

If the shape of the conductor is changed to form a loop, then the electromagnetic field around each portion of the conductor cuts across some other portion of the same conductor. This is shown in its simplest form in figure 2-2. A length of conductor is looped so that two portions of the conductor lie next to each other. These portions are labeled conductor 1 and conductor 2. When the switch is closed, current (electron flow) in the conductor produces a magnetic field around ALL portions of the conductor. For simplicity, the magnetic field (expanding lines of flux) is shown in a single plane that is perpendicular to both conductors. Although the expanding field of flux originates at the same time in both conductors, it is considered as originating in conductor 1 and its effect on conductor 2 will be explained. With increasing current, the flux field expands outward from conductor 1, cutting across a portion of conductor 2. This results in an induced emf in conductor 2 as shown by the dashed arrow. Note that the induced emf is in the opposite direction to (in OPPOSITION to) the battery current and voltage, as stated in Lenz's law.

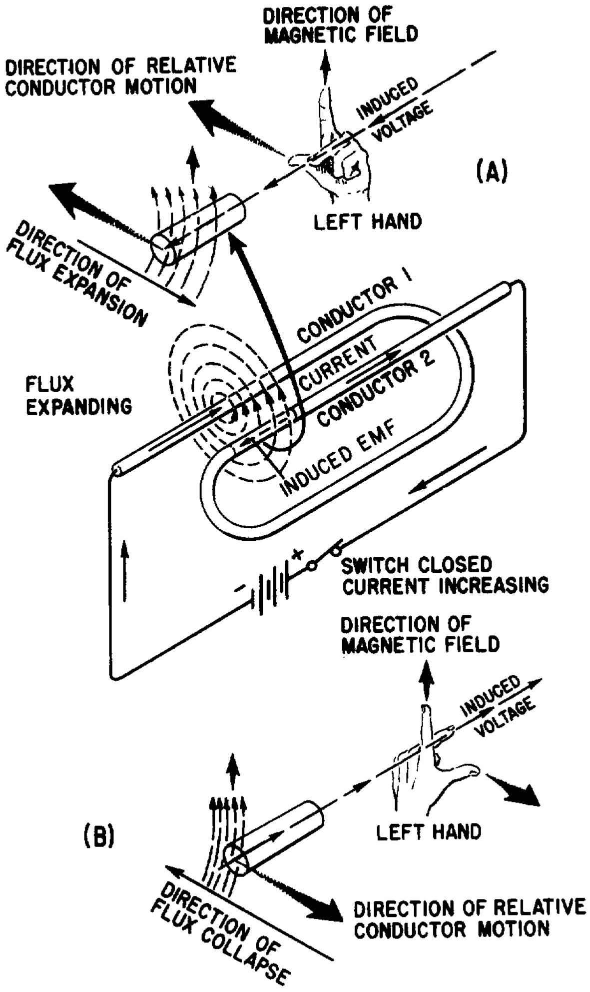


Figure 2-2.—Self-inductance.

The direction of this induced voltage may be determined by applying the LEFT-HAND RULE FOR GENERATORS. This rule is applied to a portion of conductor 2 that is "lifted" and enlarged for this purpose in figure 2-2(A). This rule states that if you point the thumb of your left hand in the direction of relative motion of the conductor and your index finger in the direction of the magnetic field, your middle finger, extended as shown, will now indicate the direction of the induced current which will generate the induced voltage (cemf) as shown.

In figure 2-2(B), the same section of conductor 2 is shown after the switch has been opened. The flux field is collapsing. Applying the left-hand rule in this case shows that the reversal of flux MOVEMENT has caused a reversal in the direction of the induced voltage. The induced voltage is now in the same direction as the battery voltage. The most important thing for you to note is that the self-induced voltage opposes BOTH changes in current. That is, when the switch is closed, this voltage delays the initial buildup of current by opposing the battery voltage. When the switch is opened, it keeps the current flowing in the same direction by aiding the battery voltage.

Thus, from the above explanation, you can see that when a current is building up it produces an expanding magnetic field. This field induces an emf in the direction opposite to the actual flow of current.

This induced emf opposes the growth of the current and the growth of the magnetic field. If the increasing current had not set up a magnetic field, there would have been no opposition to its growth. The whole reaction, or opposition, is caused by the creation or collapse of the magnetic field, the lines of which as they expand or contract cut across the conductor and develop the counter emf.

Since all circuits have conductors in them, you can assume that all circuits have inductance. However, inductance has its greatest effect only when there is a change in current. Inductance does NOT oppose current, only a CHANGE in current. Where current is constantly changing as in an ac circuit, inductance has more effect.

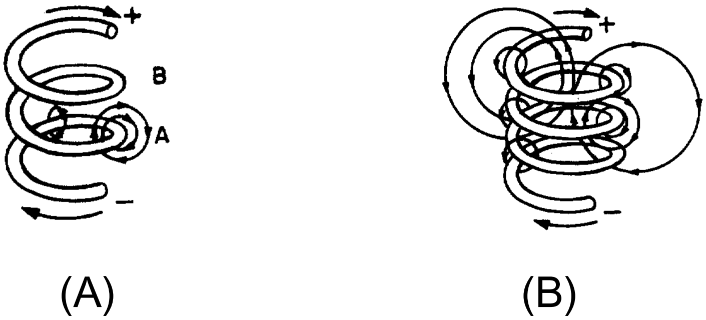
Q3. Define inductance.

Q4. What is meant by induced emf? By counter emf?

Q5. State Lenz's law.

Q6. What effect does inductance have (a) on steady direct current and (b) on direct current while it is changing in amplitude?

To increase the property of inductance, the conductor can be formed into a loop or coil. A coil is also called an inductor. Figure 2-3 shows a conductor formed into a coil. Current through one loop produces a magnetic field that encircles the loop in the direction as shown in figure 2-3(A). As current increases, the magnetic field expands and cuts all the loops as shown in figure 2-3(B). The current in each loop affects all other loops. The field cutting the other loop has the effect of increasing the opposition to a current change.



(A) (B)

Figure 2-3.—Inductance.

Inductors are classified according to core type. The core is the center of the inductor just as the core of an apple is the center of an apple. The inductor is made by forming a coil of wire around a core. The core material is normally one of two basic types: soft-iron or air. An iron-core inductor and its schematic symbol (which is represented with lines across the top of it to indicate the presence of an iron core) are shown, in figure 2-4(A). The air-core inductor may be nothing more than a coil of wire, but it is usually a coil formed around a hollow form of some nonmagnetic material such as cardboard. This material serves no purpose other than to hold the shape of the coil. An air-core inductor and its schematic symbol are shown in figure 2-4(B).

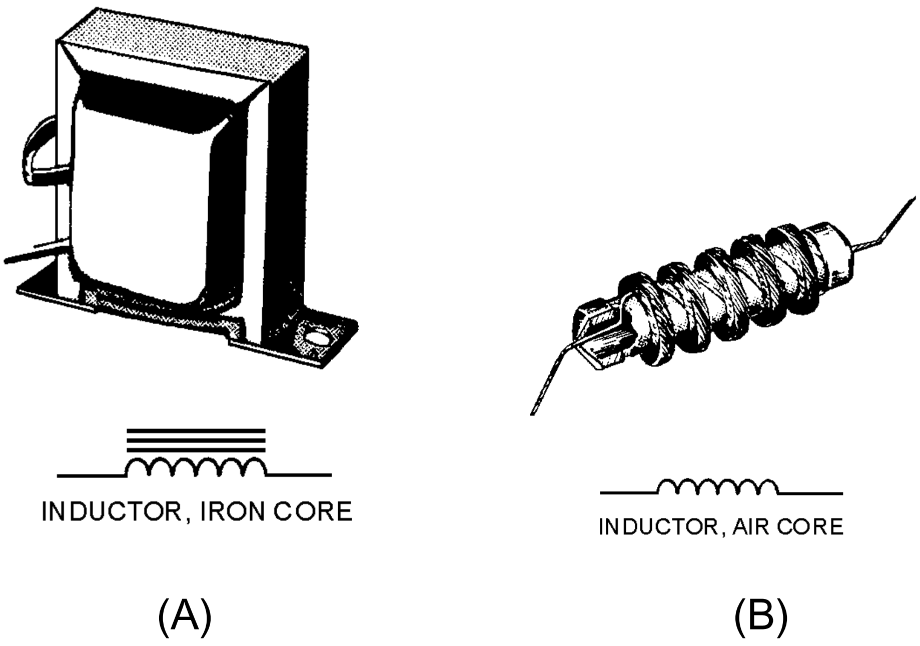


Figure 2-4.—Inductor types and schematic symbols.

Factors Affecting Coil Inductance

There are several physical factors which affect the inductance of a coil. They include the number of turns in the coil, the diameter of the coil, the coil length, the type of material used in the core, and the number of layers of winding in the coils.

Inductance depends entirely upon the physical construction of the circuit, and can only be measured with special laboratory instruments. Of the factors mentioned, consider first how the number of turns affects the inductance of a coil. Figure 2-5 shows two coils. Coil (A) has two turns and coil (B) has four turns. In coil (A), the flux field set up by one loop cuts one other loop. In coil (B), the flux field set up by one loop cuts three other loops. Doubling the number of turns in the coil will produce a field twice as strong, if the same current is used. A field twice as strong, cutting twice the number of turns, will induce four times the voltage. Therefore, it can be said that the inductance varies as the square of the number of turns.

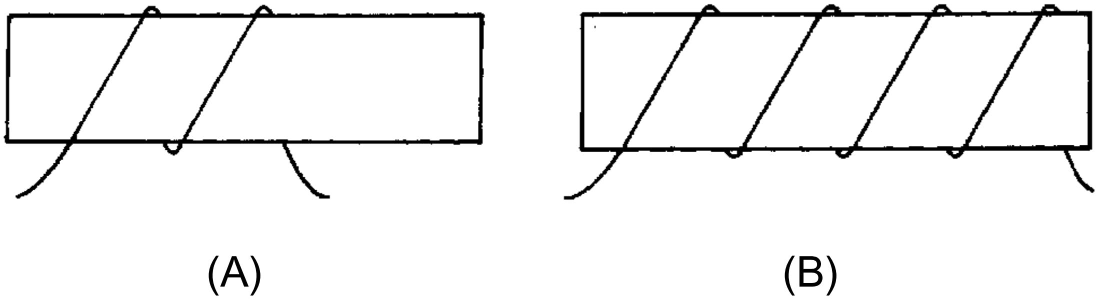
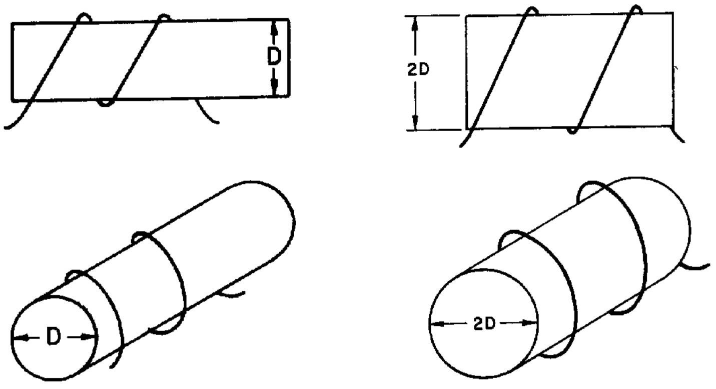


Figure 2-5.—Inductance factor (turns).

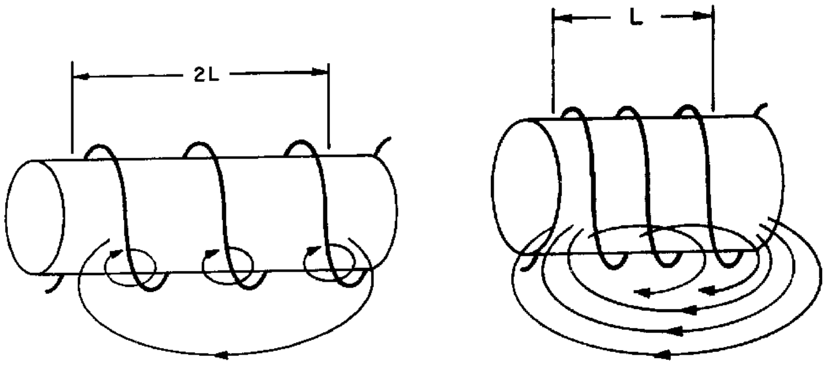
The second factor is the coil diameter. In figure 2-6 you can see that the coil in view B has twice the diameter of coil view A. Physically, it requires more wire to construct a coil of large diameter than one of small diameter with an equal number of turns. Therefore, more lines of force exist to induce a counter emf in the coil with the larger diameter. Actually, the inductance of a coil increases directly as the cross-sectional area of the core increases. Recall the formula for the area of a circle: $A = \pi r^2$. Doubling the radius of a coil increases the inductance by a factor of four.



(A) (B)

Figure 2-6.—Inductance factor (diameter).

The third factor that affects the inductance of a coil is the length of the coil. Figure 2-7 shows two examples of coil spacings. Coil (A) has three turns, rather widely spaced, making a relatively long coil. A coil of this type has few flux linkages, due to the greater distance between each turn. Therefore, coil (A) has a relatively low inductance. Coil (B) has closely spaced turns, making a relatively short coil. This close spacing increases the flux linkage, increasing the inductance of the coil. Doubling the length of a coil while keeping the same number of turns halves the value of inductance.



(A) (B)

Figure 2-7.—Inductance factor (coil length). CLOSELY WOUND

The fourth physical factor is the type of core material used with the coil. Figure 2-8 shows two coils: Coil (A) with an air core, and coil (B) with a soft-iron core. The magnetic core of coil (B) is a better path for magnetic lines of force than is the nonmagnetic core of coil (A). The soft-iron magnetic core's high permeability has less reluctance to the magnetic flux, resulting in more magnetic lines of force. This increase in the magnetic lines of force increases the number of lines of force cutting each loop of the coil, thus increasing the inductance of the coil. It should now be apparent that the inductance of a coil increases directly as the permeability of the core material increases.

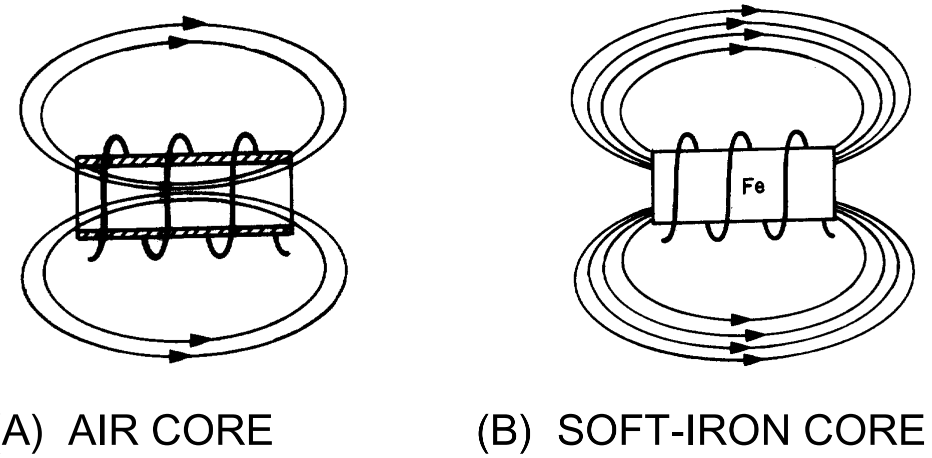


Figure 2-8.—Inductance factor (core material).

Another way of increasing the inductance is to wind the coil in layers. Figure 2-9 shows three cores with different amounts of layering. The coil in figure 2-9(A) is a poor inductor compared to the others in the figure because its turns are widely spaced and there is no layering. The flux movement, indicated by the dashed arrows, does not link effectively because there is only one layer of turns. A more inductive coil is shown in figure 2-9(B). The turns are closely spaced and the wire has been wound in two layers. The two layers link each other with a greater number of flux loops during all flux movements. Note that nearly all the turns, such as X, are next to four other turns (shaded). This causes the flux linkage to be increased.

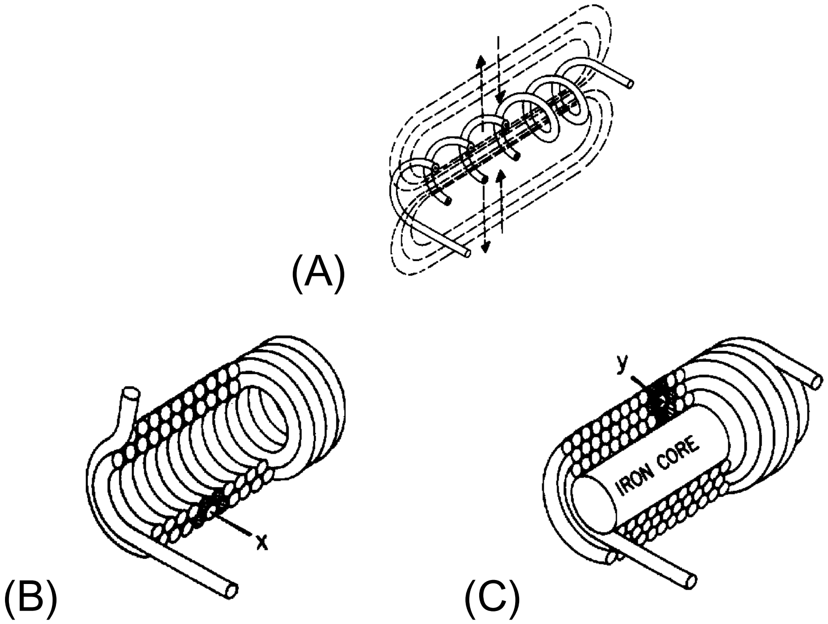


Figure 2-9.—Coils of various inductances.

A coil can be made still more inductive by winding it in three layers, as shown in figure 2-9(C). The increased number of layers (cross-sectional area) improves flux linkage even more. Note that some turns, such as Y, lie directly next to six other turns (shaded). In actual practice, layering can continue on through many more layers. The important fact to remember, however, is that the inductance of the coil increases with each layer added.

As you have seen, several factors can affect the inductance of a coil, and all of these factors are variable. Many differently constructed coils can have the same inductance. The important information to remember, however, is that inductance is dependent upon the degree of linkage between the wire conductor(s) and the electromagnetic field. In a straight length of conductor, there is very little flux linkage between one part of the conductor and another. Therefore, its inductance is extremely small. It was shown that conductors become much more inductive when they are wound into coils. This is true because there is maximum flux linkage between the conductor turns, which lie side by side in the coil.

Q7.

a. *List five factors that affect the inductance of a coil.*

b. *Bending a straight piece of wire into a loop or coil has what effect on the inductance of the wire?*

c. *Doubling the number of turns in a coil has what effect on the inductance of the coil?*

d. *Decreasing the diameter of a coil has what effect on the inductance of the coil?*

e. *Inserting a soft-iron core into a coil has what effect on the inductance of the coil?*

f. *Increasing the number of layers of windings in a coil has what effect on the inductance of the coil?*

UNIT OF INDUCTANCE

As stated before, the basic unit of inductance (L) is the HENRY (H), named after Joseph Henry, the co-discoverer with Faraday of the principle of electromagnetic induction. An inductor has an inductance of 1 henry if an emf of 1 volt is induced in the inductor when the current through the inductor is changing at the rate of 1 ampere per second. The relationship between the induced voltage, the inductance, and the rate of change of current with respect to time is stated mathematically as:

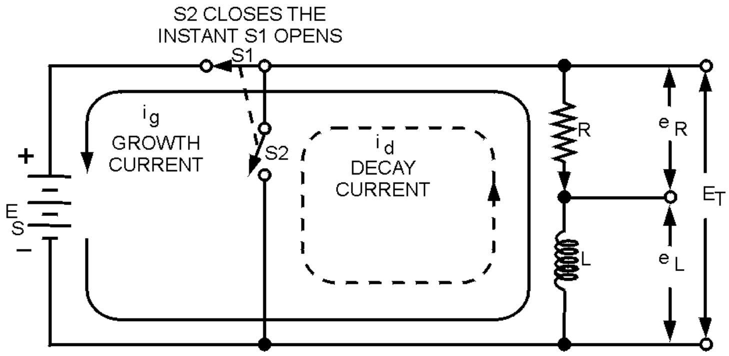
$$E_{ind} = L\frac{\Delta I}{\Delta t}$$

where E_{ind} is the induced emf in volts; L is the inductance in henrys; and ΔI is the change in current in amperes occurring in Δt seconds. The symbol Δ (Greek letter delta), means "a change in ....". The henry is a large unit of inductance and is used with relatively large inductors. With small inductors, the millihenry is used. (A millihenry is equal to 1×10^{-3} henry, and one henry is equal to 1,000 millihenrys.) For still smaller inductors the unit of inductance is the microhenry (µH). (A $\mu H = 1 \times 10^{-6} H$, and one henry is equal to 1,000,000 microhenrys.)

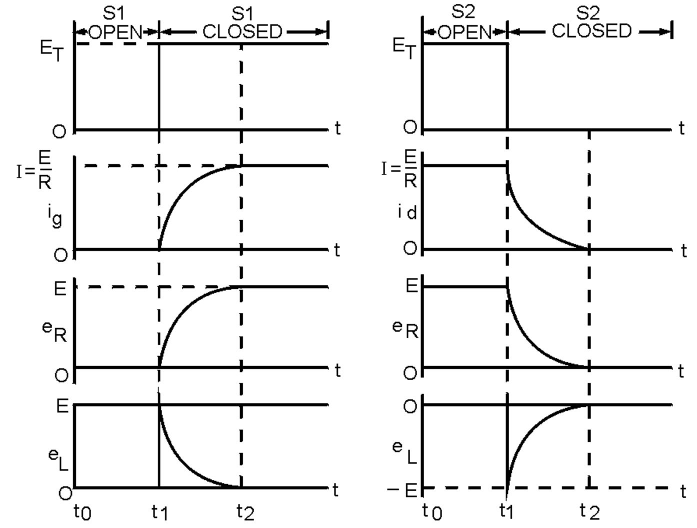
GROWTH AND DECAY OF CURRENT IN AN LR SERIES CIRCUIT

When a battery is connected across a "pure" inductance, the current builds up to its final value at a rate determined by the battery voltage and the internal resistance of the battery. The current buildup is gradual because of the counter emf generated by the self-inductance of the coil. When the current starts to flow, the magnetic lines of force move outward from the coil. These lines cut the turns of wire on the inductor and build up a counter emf that opposes the emf of the battery. This opposition causes a delay in the time it takes the current to build up to a steady value. When the battery is disconnected, the lines of force collapse. Again these lines cut the turns of the inductor and build up an emf that tends to prolong the flow of current.

A voltage divider containing resistance and inductance may be connected in a circuit by means of a special switch, as shown in figure 2-10(A). Such a series arrangement is called an LR series circuit.



CIRCUIT
(A)



GROWTH
(B)

DECAY
(C)

Figure 2-10.—Growth and decay of current in an LR series circuit.

When switch S_1 is closed (as shown), a voltage E_S appears across the voltage divider. At this instant the current will attempt to increase to its maximum value. However, this instantaneous current change causes coil L to produce a back EMF, which is opposite in polarity and almost equal to the EMF of the source. This back EMF opposes the rapid current change. Figure 2-10(B) shows that at the instant switch S_1 is closed, there is no measurable growth current (i_g), a minimum voltage drop is across resistor R, and maximum voltage exists across inductor L.

As current starts to flow, a voltage (e_R) appears across R, and the voltage across the inductor is reduced by the same amount. The fact that the voltage across the inductor (L) is reduced means that the growth current (i_g) is increased and consequently e_R is increased. Figure 2-10(B) shows that the voltage across the inductor (e_L) finally becomes zero when the growth current (i_g) stops increasing, while the voltage across the resistor (e_R) builds up to a value equal to the source voltage (E_S).

Electrical inductance is like mechanical inertia, and the growth of current in an inductive circuit can be likened to the acceleration of a boat on the surface of the water. The boat does not move at the instant a constant force is applied to it. At this instant all the applied force is used to overcome the inertia of the boat. Once the inertia is overcome the boat will start to move. After a while, the speed of the boat reaches its maximum value and the applied force is used up in overcoming the friction of the water against the hull.

When the battery switch (S_1) in the LR circuit of figure 2-10(A) is closed, the rate of the current increase is maximum in the inductive circuit. At this instant all the battery voltage is used in overcoming the emf of self-induction which is a maximum because the rate of change of current is maximum. Thus the battery voltage is equal to the drop across the inductor and the voltage across the resistor is zero. As time goes on more of the battery voltage appears across the resistor and less across the inductor. The rate of change of current is less and the induced emf is less. As the steady-state condition of the current is approached, the drop across the inductor approaches zero and all of the battery voltage is "dropped" across the resistance of the circuit.

Thus the voltages across the inductor and the resistor change in magnitude during the period of growth of current the same way the force applied to the boat divides itself between the effects of inertia and friction. In both examples, the force is developed first across the inertia/inductive effect and finally across the friction/resistive effect.

Figure 2-10(C) shows that when switch S_2 is closed (source voltage E_S removed from the circuit), the flux that has been established around the inductor (L) collapses through the windings. This induces a voltage e_L in the inductor that has a polarity opposite to E_S and is essentially equal to E_S in magnitude. The induced voltage causes decay current (i_d) to flow in resistor R in the same direction in which current was flowing originally (when S_1 was closed). A voltage (e_R) that is initially equal to source voltage (E_S) is developed across R. The voltage across the resistor (e_R) rapidly falls to zero as the voltage across the inductor (e_L) falls to zero due to the collapsing flux.

Just as the example of the boat was used to explain the growth of current in a circuit, it can also be used to explain the decay of current in a circuit. When the force applied to the boat is removed, the boat still continues to move through the water for a while, eventually coming to a stop. This is because energy was being stored in the inertia of the moving boat. After a period of time the friction of the water overcomes the inertia of the boat, and the boat stops moving. Just as inertia of the boat stored energy, the magnetic field of an inductor stores energy. Because of this, even when the power source is removed, the stored energy of the magnetic field of the inductor tends to keep current flowing in the circuit until the magnetic field collapse.

Q8.

a. *When voltage is first applied to a series LR circuit, how much opposition does the inductance have to the flow of current compared to that of the circuit resistance?*

b. *In a series circuit containing a resistor (R_1) and an inductor (L_1), what voltage exists across R_1 when the counter emf is at its maximum value?*

c. *What happens to the voltage across the resistance in an LR circuit during current buildup in the circuit, and during current decay in the circuit?*

L/R Time Constant

The L/R TIME CONSTANT is a valuable tool for use in determining the time required for current in an inductor to reach a specific value. As shown in figure 2-11, one L/R time constant is the time required for the current in an inductor to increase to 63 percent (actually 63.2 percent) of the maximum current. Each time constant is equal to the time required for the current to increase by 63.2 percent of the difference in value between the current flowing in the inductor and the maximum current. Maximum current flows in the inductor after five L/R time constants are completed. The following example should clear up any confusion about time constants. Assume that maximum current in an LR circuit is 10 amperes. As you know, when the circuit is energized, it takes time for the current to go from zero to 10 amperes. When the first time constant is completed, the current in the circuit is equal to 63.2% of 10 amperes. Thus the amplitude of current at the end of 1 time constant is 6.32 amperes.

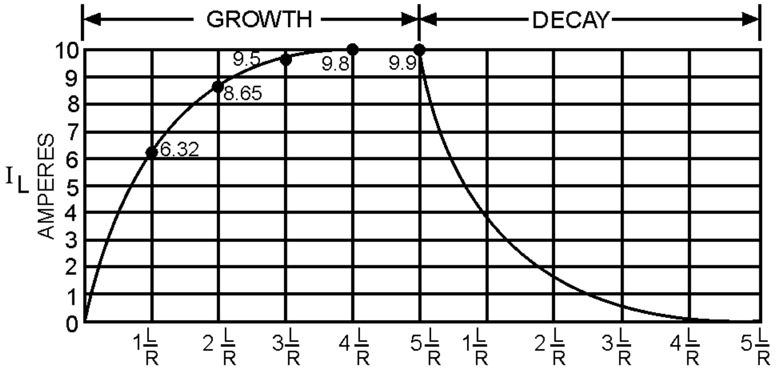


Figure 2-11.—L/R time constant.

During the second time constant, current again increases by 63.2% (.632) of the difference in value between the current flowing in the inductor and the maximum current. This difference is 10 amperes minus 6.32 amperes and equals 3.68 amperes; 63.2% of 3.68 amperes is 2.32 amperes. This increase in current during the second time constant is added to that of the first time constant. Thus, upon completion of the second time constant, the amount of current in the LR circuit is 6.32 amperes + 2.32 amperes = 8.64 amperes.

During the third constant, current again increases:

$$\begin{aligned} 10\text{ amperes} - 8.64\text{ amperes} &= 1.36\text{ amperes} \\ 1.36\text{ amperes} \times .632 &= 0.860\text{ ampere} \\ 8.64\text{ amperes} + 0.860\text{ ampere} &= 9.50\text{ amperes} \end{aligned}$$

During the fourth time constant, current again increases:

$$\begin{aligned} 10\text{ amperes} - 9.50\text{ amperes} &= 0.5\text{ ampere} \\ 0.5\text{ ampere} \times .632 &= 0.316\text{ ampere} \\ 9.50\text{ amperes} + 0.316\text{ ampere} &= 9.82\text{ amperes} \end{aligned}$$

During the fifth time constant, current increases as before:

$$\begin{aligned} 10 \text{ amperes} - 9.82 \text{ amperes} &= 0.18 \text{ ampere} \\ 0.18 \text{ ampere} \times .632 &= 0.114 \text{ ampere} \\ 9.82 \text{ amperes} + .114 \text{ ampere} &= 9.93 \text{ amperes} \end{aligned}$$

Thus, the current at the end of the fifth time constant is almost equal to 10.0 amperes, the maximum current. For all practical purposes the slight difference in value can be ignored.

When an LR circuit is deenergized, the circuit current decreases (decays) to zero in five time constants at the same rate that it previously increased. If the growth and decay of current in an LR circuit are plotted on a graph, the curve appears as shown in figure 2-11. Notice that current increases and decays at the same rate in five time constants.

The value of the time constant in seconds is equal to the inductance in henrys divided by the circuit resistance in ohms.

The formula used to calculate one L/R time constant is:

$$\text{Time Constant (TC) in seconds} = \frac{\text{L (in henrys)}}{\text{R (in ohms)}}$$

Q9. What is the formula for one L/R time constant?

Q10.

a. The maximum current applied to an inductor is 1.8 amperes. How much current flowed in the inductor 3 time constants after the circuit was first energized?

b. What is the minimum number of time constants required for the current in an LR circuit to increase to its maximum value?

c. A circuit containing only an inductor and a resistor has a maximum of 12 amperes of applied current flowing in it. After 5 L/R time constants the circuit is opened. How many time constants is required for the current to decay to 1.625 amperes?

POWER LOSS IN AN INDUCTOR

Since an inductor (coil) consists of a number of turns of wire, and since all wire has some resistance, every inductor has a certain amount of resistance. Normally this resistance is small. It is usually neglected in solving various types of ac circuit problems because the reactance of the inductor (the opposition to alternating current, which will be discussed later) is so much greater than the resistance that the resistance has a negligible effect on the current.

However, since some inductors are designed to carry relatively large amounts of current, considerable power can be dissipated in the inductor even though the amount of resistance in the inductor is small. This power is wasted power and is called COPPER LOSS. The copper loss of an inductor can be calculated by multiplying the square of the current in the inductor by the resistance of the winding (I^2R).

In addition to copper loss, an iron-core coil (inductor) has two iron losses. These are called HYSTERESIS LOSS and EDDY-CURRENT LOSS. Hysteresis loss is due to power that is consumed in reversing the magnetic field of the inductor core each time the direction of current in the inductor changes.

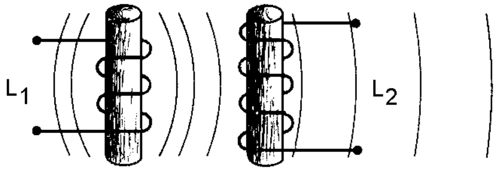
Eddy-current loss is due to heating of the core by circulating currents that are induced in the iron core by the magnetic field around the turns of the coil. These currents are called eddy currents and circulate within the iron core only.

All these losses dissipate power in the form of heat. Since this power cannot be returned to the electrical circuit, it is lost power.

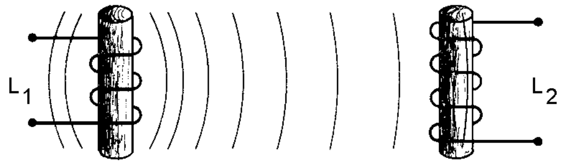
Q11. State three types of power loss in an inductor.

MUTUAL INDUCTANCE

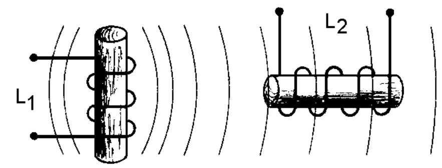
Whenever two coils are located so that the flux from one coil links with the turns of the other coil, a change of flux in one coil causes an emf to be induced in the other coil. This allows the energy from one coil to be transferred or coupled to the other coil. The two coils are said to be coupled or linked by the property of MUTUAL INDUCTANCE (M). The amount of mutual inductance depends on the relative positions of the two coils. This is shown in figure 2-12. If the coils are separated a considerable distance, the amount of flux common to both coils is small and the mutual inductance is low. Conversely, if the coils are close together so that nearly all the flux of one coil links the turns of the other, the mutual inductance is high. The mutual inductance can be increased greatly by mounting the coils on a common iron core.



(A) INDUCTORS CLOSE – LARGE M



(B) INDUCTORS FAR APART - SMALL M



(C) INDUCTOR AXES PERPENDICULAR – NO M

Figure 2-12.—The effect of position of coils on mutual inductance (M).

Two coils are placed close together as shown in figure 2-13. Coil 1 is connected to a battery through switch S, and coil 2 is connected to an ammeter (A). When switch S is closed as in figure 2-13(A), the current that flows in coil 1 sets up a magnetic field that links with coil 2, causing an induced voltage in coil 2 and a momentary deflection of the ammeter. When the current in coil 1 reaches a steady value, the ammeter returns to zero. If switch S is now opened as in figure 2-13(B), the ammeter (A) deflects momentarily in the opposite direction, indicating a momentary flow of current in the opposite direction in coil 2. This current in coil 2 is produced by the collapsing magnetic field of coil 1.

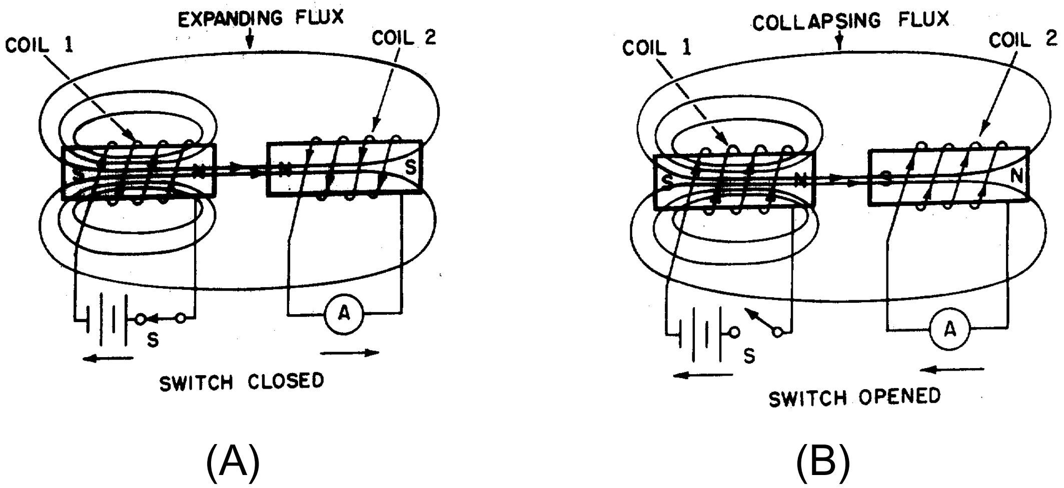


Figure 2-13.—Mutual inductance.

Q12. Define mutual inductance.

FACTORS AFFECTING MUTUAL INDUCTANCE

The mutual inductance of two adjacent coils is dependent upon the physical dimensions of the two coils, the number of turns in each coil, the distance between the two coils, the relative positions of the axes of the two coils, and the permeability of the cores.

The COEFFICIENT OF COUPLING between two coils is equal to the ratio of the flux cutting one coil to the flux originated in the other coil. If the two coils are so positioned with respect to each other so that all of the flux of one coil cuts all of the turns of the other, the coils are said to have a unity coefficient of coupling. It is never exactly equal to unity (1), but it approaches this value in certain types of coupling devices. If all of the flux produced by one coil cuts only half the turns of the other coil, the coefficient of coupling is 0.5. The coefficient of coupling is designated by the letter K.

The mutual inductance between two coils, L_1 and L_2, is expressed in terms of the inductance of each coil and the coefficient of coupling K. As a formula:

$$M = K\sqrt{L_1 L_2}$$

where: M = Mutual inductance in henrys

K = Coefficient of coupling

L_1, L_2 = Inductance of coil in henrys

Example problem:

One 10-H coil and one 20-H coil are connected in series and are physically close enough to each other so that their coefficient of coupling is 0.5. What is the mutual inductance between the coils?

Use the formula: $M = K\sqrt{L_1 L_2}$

$$M = 0.5\sqrt{(10H)(20H)}$$

$$M = 0.5\sqrt{200H}$$

$$M = 0.5 \times 14.14H$$

$$M = 7.07H$$

Q13. When are two circuits said to be coupled?

Q14. What is meant by the coefficient of coupling?

SERIES INDUCTORS WITHOUT MAGNETIC COUPLING

When inductors are well shielded or are located far enough apart from one another, the effect of mutual inductance is negligible. If there is no mutual inductance (magnetic coupling) and the inductors are connected in series, the total inductance is equal to the sum of the individual inductances. As a formula:

$$L_T = L_1 + L_2 + L_3 + .. L_n$$

where L_T is the total inductance; L_1, L_2, L_3 are the inductances of L_1, L_2, L_3; and L_n means that any number (n) of inductors may be used. The inductances of inductors in series are added together like the resistances of resistors in series.

SERIES INDUCTORS WITH MAGNETIC COUPLING

When two inductors in series are so arranged that the field of one links the other, the combined inductance is determined as follows:

$$L_T = L_1 + L_2 \pm 2M$$

where: L_T = The total inductance

L_1, L_2 = The inductances of L_1, L_2

M = The mutual inductance between the two inductors

The plus sign is used with M when the magnetic fields of the two inductors are aiding each other, as shown in figure 2-14. The minus sign is used with M when the magnetic field of the two inductors oppose each other, as shown in figure 2-15. The factor 2M accounts for the influence of L_1 on L_2 and L_2 on L_1.

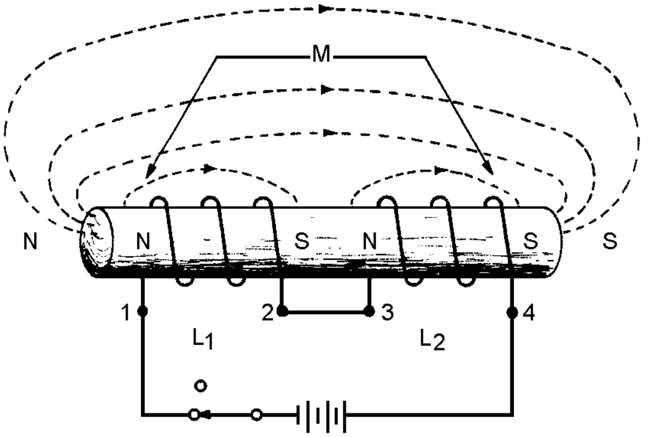


Figure 2-14.—Series inductors with aiding fields.

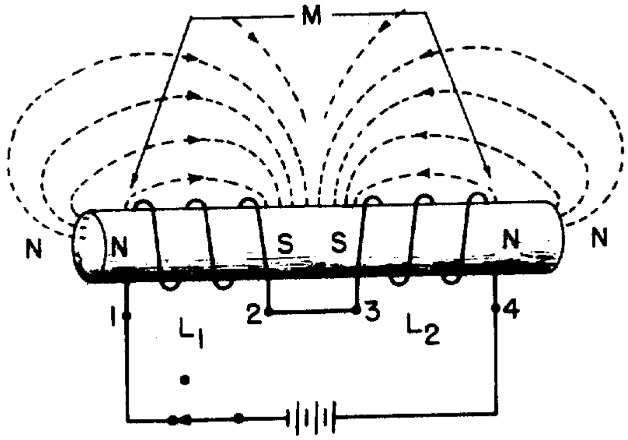


Figure 2-15.—Series inductors with opposing fields.

Example problem:

A 10-H coil is connected in series with a 5-H coil so the fields aid each other. Their mutual inductance is 7 H. What is the combined inductance of the coils?

Use the formula:
$$L_T = L_1 + L_2 + 2M$$
$$L_T = 10\ H + 5\ H + 2(7\ H)$$
$$L_T = 29\ H$$

Q15. Two series-connected 7-H inductors are adjacent to each other; their coefficient of coupling is 0.64. What is the value of M?

Q16. A circuit contains two series inductors aligned in such a way that their magnetic fields oppose each other. What formula should you use to compute total inductance in this circuit?

Q17. The magnetic fields of two coils are aiding each other. The inductance of the coils are 3 H and 5 H, respectively. The coils' mutual inductance is 5 H. What is their combined inductance?

PARALLEL INDUCTORS WITHOUT COUPLING

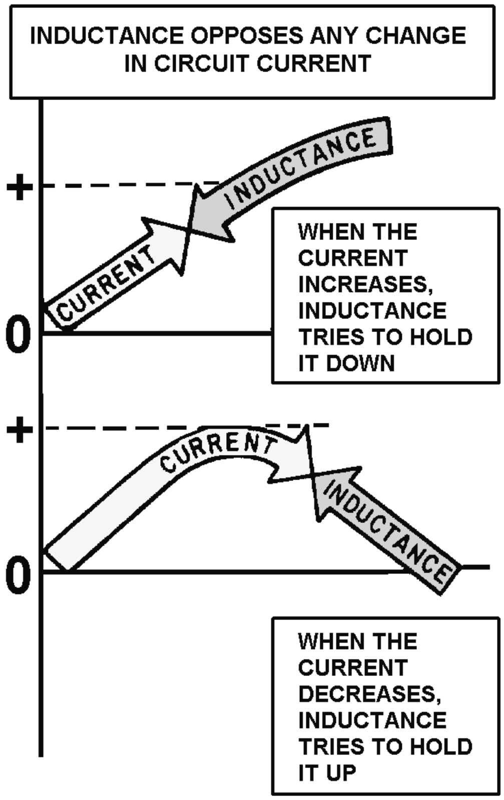
The total inductance (L_T) of inductors in parallel is calculated in the same manner that the total resistance of resistors in parallel is calculated, provided the coefficient of coupling between the coils is zero. Expressed mathematically:

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} .... + \frac{1}{L_N}$$

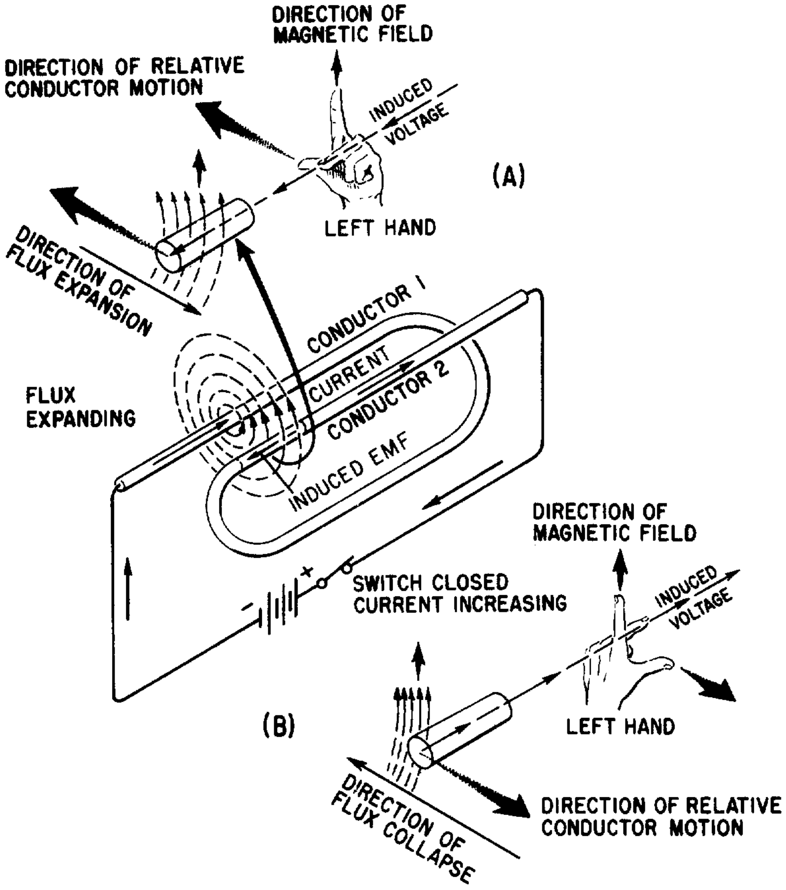
SUMMARY

The important points of this chapter are summarized below. Study this information before continuing, as this information will lay the foundation for later chapters.

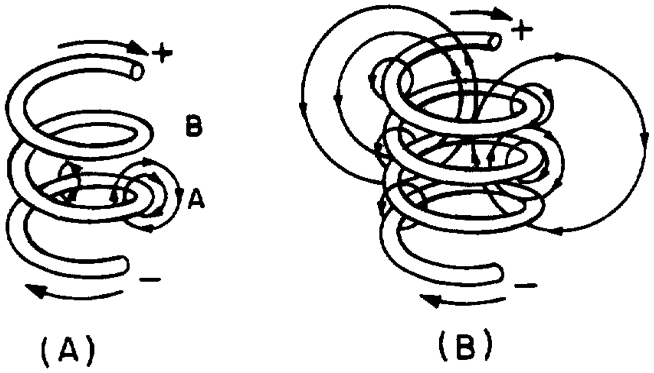
INDUCTANCE—The characteristic of an electrical circuit that opposes a change in current. The reaction (opposition) is caused by the creation or destruction of a magnetic field. When current starts to flow, magnetic lines of force are created. These lines of force cut the conductor inducing a counter emf in a direction that opposes current.



SELF-INDUCTANCE—The process by which a circuit induces an emf into itself by its own moving magnetic field. All electrical circuits possess self-inductance. This opposition (inductance), however, only takes place when there is a change in current. Inductance does NOT oppose current, only a CHANGE in current. The property of inductance can be increased by forming the conductor into a loop. In a loop, the magnetic lines of force affect more of the conductor at one time. This increases the self-induced emf.

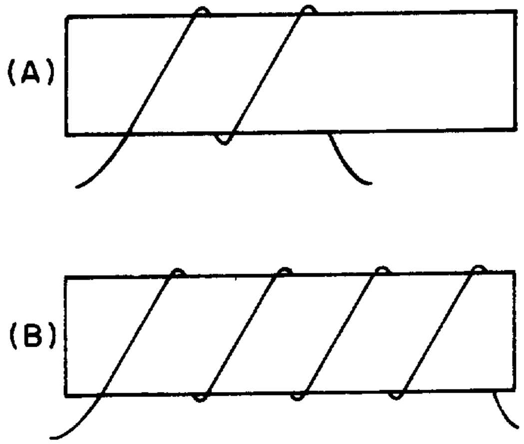


INDUCTANCE OF A COIL—The property of inductance can be further increased if the conductor is formed into a coil. Because a coil contains more loops, more of the conductor can be affected by the magnetic field. Inductors (coils) are classified according to core type. The core material is normally either air or soft iron.

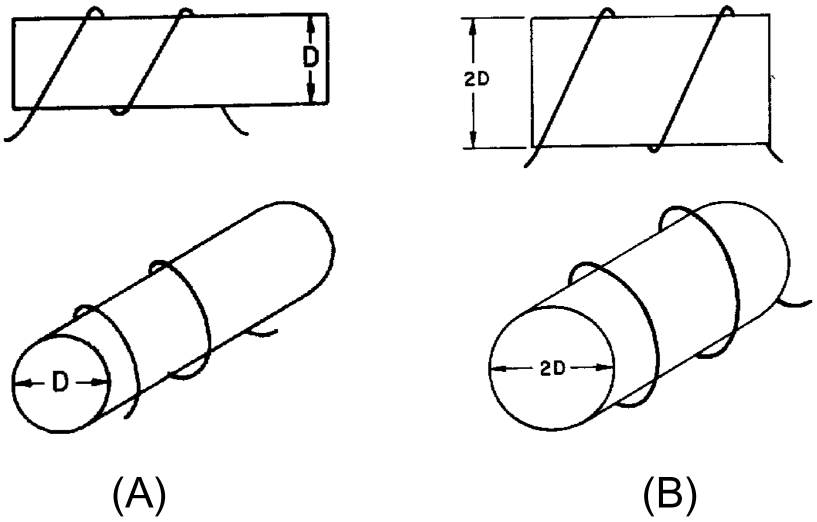


FACTORS AFFECTING COIL INDUCTANCE—The inductance of a coil is entirely dependent upon its physical construction. Some of the factors affecting the inductance are:

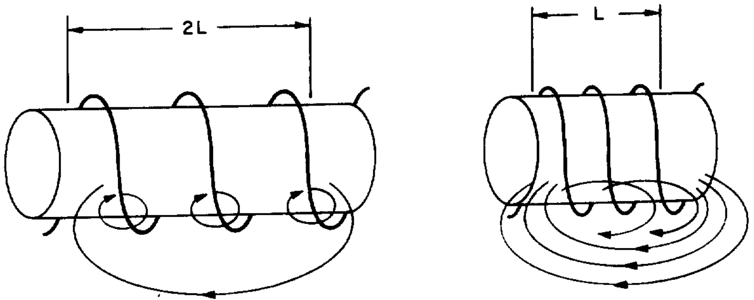
- The number of turns in the coil. Increasing the number of turns will increase the inductance.



- The coil diameter. The inductance increases directly as the cross-sectional area of the coil increases.



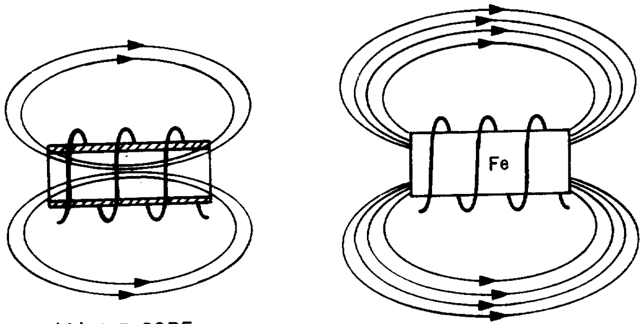
- The length of the coil. When the length of the coil is increased while keeping the number of turns the same, the turn-spacing is increased. This decreases the inductance of the coil.



(A) WIDELY SPACED

(B) CLOSELY WOUND

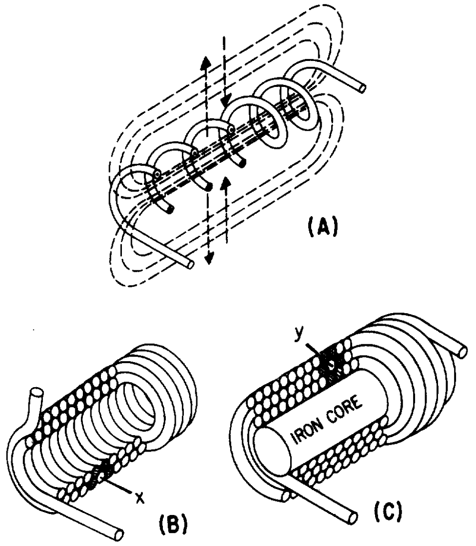
- The type of core material. Increasing the permeability of the core results in increasing the inductance of the coil.



(A) AIR CORE

(B) SOFT-IRON CORE

- Winding the coil in layers. The more layers used to form a coil, the greater effect the magnetic field has on the conductor. By layering a coil, you can increase the inductance.



UNIT OF INDUCTANCE—Inductance (L) is measured in henrys (H). An inductor has an inductance of one henry (H) if an emf of one volt is induced in the inductor when the current through the inductor is changing at the rate of 1 ampere per second. Common units of inductance are henry (H), millihenry (mH) and the microhenry (µH).

GROWTH AND DECAY OF CURRENT IN AN LR CIRCUIT—The required for the current in an inductor to increase to 63.2 percent of the maximum current or to decrease to 36.8 percent of the maximum current is known as the time constant. The letter symbol for an LR time constant is L/R. As a formula:

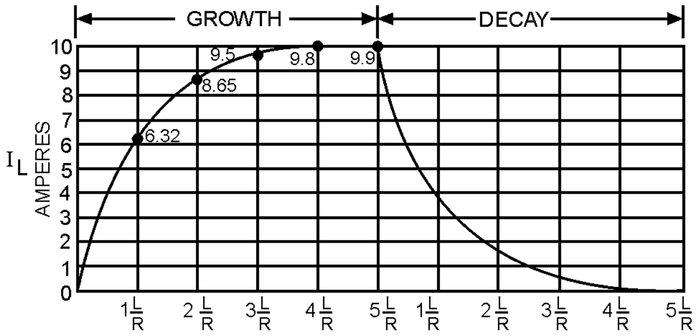
$$\frac{L}{R}.$$

As a formula:

$$t \text{ (in seconds)} = \frac{L \text{ (in henrys)}}{R \text{ (in ohms)}}$$

or

$$t \text{ (in microseconds)} = \frac{L \text{ (in microhenrys)}}{R \text{ (in ohms)}}$$

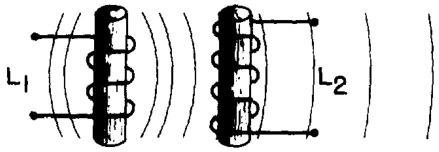


The time constant of an LR circuit may also be defined as the time required for the current in the inductor to grow or decay to its final value if it continued to grow or decay at its initial rate. For all practical purposes, the current in the inductor reaches a maximum value in 5 "Time Constants" and decreases to zero in 5 "Time Constants".

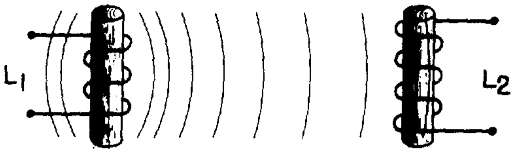
POWER LOSSES IN AN INDUCTOR—Since an inductor (coil) contains a number turns of wire, and all wire has some resistance, the inductor has a certain amount of resistance. This resistance is usually very small and has a negligible effect on current. However, there are power losses in an inductor. The main power losses in an inductor are copper loss, hysteresis loss, and eddy-current loss. Copper loss can be calculated by multiplying the square of the current by the resistance of the wire in the coil (I_2R). Hysteresis loss is due to power that is consumed in reversing the magnetic field of the core each time the current direction changes. Eddy-current loss is due to core heating caused by circulating currents induced in an iron core by the magnetic field of the coil.

MUTUAL INDUCTANCE—When two coils are located so that the flux from one coil cuts the turns of the other coil, the coils have mutual inductance. The amount of mutual inductance depends upon several factors: the relative position of the axes of the two coils; the permeability of the cores; the physical dimensions of the two coils; the number of turns in each coil, and the distance between the coils. The coefficient of coupling K specifies the amount of coupling between the coils. If all of the flux from one coil cuts all of the turns of the other coil, the coefficient of coupling K is 1 or unity. If none of the flux from one coil cuts the turns of the other coil, the coefficient of coupling is zero. The mutual inductance between two coils (L_1 and L_2) may be expressed mathematically as:

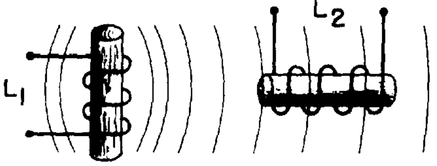
$$M = K\sqrt{L_1 L_2}$$



(A) INDUCTORS CLOSE -- LARGE M



(B) INDUCTORS FAR APART -- SMALL M



(C) INDUCTOR AXES PERPENDICULAR -- NO M

<u>COMPUTING THE INDUCTANCE OF A CIRCUIT</u>—When the total inductance of a circuit is computed, the individual inductive values are treated the same as resistance values. The inductances of inductors in series are added like the resistances of resistors in series. That is,

$$L_T = L_1 + L_2 + L_3 \ldots + L_n$$

The inductances of inductors in parallel are combined mathematically like the resistances of resistors in parallel. That is,

$$L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots \frac{1}{L_n}}$$

Both of the above formulas are accurate, providing there is no mutual inductance between the inductors.

ANSWERS TO QUESTIONS Q1. THROUGH Q17.

A1. *The henry, H.*

A2. *Magnetic field.*

A3. *Inductance is the property of a coil (or circuit) which opposes any CHANGE in current.*

A4. *Induced emf is the emf which appears across a conductor when there is relative motion between the conductor and a magnetic field; counter emf is the emf induced in a conductor that opposes the applied voltage.*

A5. *The induced emf in any circuit is in a direction to oppose the effect that produced it.*

A6.

- *a.* *No effect.*
- *b.* *Inductance opposes any change in the amplitude of current.*

A7.

- a.
  - *1.* *The numbers of turns in a coil.*
  - *2.* *The type of material used in the core.*
  - *3.* *The diameter of the coil.*
  - *4.* *The coil length.*
  - *5.* *The number of layers of windings in the coil.*
- *b.* *Increases inductance.*
- *c.* *Increases inductance.*
- *d.* *Decreases inductance.*
- *e.* *Increases inductance.*
- *f.* *Increases inductance.*

A8.

- *a.* *Inductance causes a very large opposition to the flow of current when voltage is first applied to an LR circuit; resistance causes comparatively little opposition to current at that time.*
- *b.* *Zero.*
- *c.* *During current buildup, the voltage across the resistor gradually increases to the same voltage as the source voltage; and during current decay the voltage across the resistor gradually drops to zero*

A9.

$$t = \frac{L}{R}$$

A10.

a. 1.71 amperes.

b. 5 time constants.

c. 2 time constants.

A11. Copper loss; hysteresis loss; eddy-current loss.

A12. Mutual inductance is the property existing between two coils so positioned that flux from one coil cuts the windings of the other coil.

A13. When they are arranged so that energy from one circuit is transferred to the other circuit.

A14. The ratio of the fines of force produced by one coil to the lines of force that link another coil. It is never greater than one.

A15.

$$4.48\text{ H (because } M = K\sqrt{7H \times 7H} = 0.64 \times 7H = 4.48H)$$

A16.

$$L_T = L_1 + L_2 - 2M$$

A17.

$$\begin{aligned} L_T = 18\text{ H (because } L_T &= L_1 + L_2 + 2M \\ L_T &= 3H + 5H + 2(5H) = 18H) \end{aligned}$$